

# No Drama Quantum Electrodynamics?

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## Abstract

After introduction of a complex electromagnetic potential, which leaves the electromagnetic fields unchanged, the spinor field can be eliminated from the equations of spinor electrodynamics (the Dirac-Maxwell electrodynamics), and the resulting system of equations for the complex four-potential describes independent evolution of electromagnetic field in the following sense: if components of the complex four-potential and their derivatives with respect to time up to the second order are known at some point in time in the entire 3D space, the equations determine the third-order derivatives of the components with respect to time, so the Cauchy problem can be posed, and the equations can be integrated (at least locally). This result permits mathematical simplification and can be useful for interpretation of quantum theory. A generalized Carleman linearization procedure generates a system of linear equations in the Fock space, which looks like a second-quantized theory and is equivalent to the original nonlinear system on the set of solutions of the latter.

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## I. INTRODUCTION

This work builds on the results of article [1] (see also Ref. [2] and Ref. [3]). To show how the mathematical result of this work is physically relevant and make the work reasonably self-contained, it is necessary to repeat or rephrase some argumentation of Ref. [1].

”No drama” quantum theory is informally defined in Ref. [1] as ”something as simple (in principle) as classical electrodynamics - a local realistic theory described by a system of partial differential equations in 3+1 dimensions, but reproducing unitary evolution of quantum theory in the configuration space”. The issue of inconsistency with the Bell theorem is discussed in Sec. 5 of Ref. [1]) using other people’s arguments, and the discussion is fully applicable to this work, but is not reproduced here (briefly: the new theory adopts unitary evolution of standard quantum theory, but not its theory of measurement, which is notoriously problematic anyway).

It was shown in Ref. [1]) that the matter field can be naturally eliminated from the equations of scalar electrodynamics (the Klein-Gordon-Maxwell electrodynamics) in the unitary gauge, and the resulting equations describe independent evolution of the electromagnetic field. For these nonlinear partial differential equations, a generalized Carleman linearization procedure is used (following nightlight’s observation) to generate a system of linear equations in the Fock space, which looks like a second-quantized theory and is equivalent to the original nonlinear system on the set of solutions of the latter.

While spinor electrodynamics (the Dirac-Maxwell electrodynamics) is clearly preferable to scalar electrodynamics, the generalization of the above results to spinor electrodynamics in Ref. [1]) is much less complete and satisfactory. The aim of this work is to overcome this weakness and to offer a new generalization of the results to spinor electrodynamics. To this end, a complex four-potential of electromagnetic field [4] is introduced, but it produces the same real fields, as it differs from a real four-potential by a gradient. It is shown that for each solution of spinor electrodynamics the spinor can be eliminated in a certain generalized gauge, and the resulting equations for the complex four-potential describe independent evolution of the electrodynamic field. An apparently second-quantized theory equivalent to the equations of this evolution on the set of solutions of the latter can be built in the same way as in Ref. [1]. Comparison of the resulting theory with standard quantum electrodynamics and experimental data is beyond the scope of this work. However, this

theory presents at least an important toy model of quantum electrodynamics. Furthermore, the mathematical result of this work is valid for the important and realistic theory – spinor electrodynamics.

## II. ELIMINATION OF THE SPINOR FIELD

The following derivation builds on the result of article [5], where three out of four components of the spinor function are eliminated from the Dirac equation in a general case, yielding an equivalent fourth-order partial differential equation. Furthermore, the remaining component can be made real by a gauge transform.

Let us consider a solution of the equations of (non-second-quantized) spinor electrodynamics  $A^\mu$ ,  $\psi$ :

$$(i\cancel{\partial} - \cancel{A})\psi = \psi, \quad (1)$$

$$\square A_\mu - A^\nu_{,\nu\mu} = e^2 \bar{\psi} \gamma_\mu \psi, \quad (2)$$

where, e.g.,  $\cancel{A} = A_\mu \gamma^\mu$  (the Feynman slash notation). For the sake of simplicity, a system of units is used where  $\hbar = c = m = 1$ , and the electric charge  $e$  is included in  $A_\mu$  ( $eA_\mu \rightarrow A_\mu$ ). In the chiral representation of  $\gamma$ -matrices (Ref. [6])

$$\gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}, \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad (3)$$

where index  $i$  runs from 1 to 3, and  $\sigma^i$  are the Pauli matrices.

Let us apply the following "generalized gauge transform":

$$\psi = \exp(i\alpha)\varphi, \quad (4)$$

$$A_\mu = B_\mu - \alpha_{,\mu}, \quad (5)$$

where the new four-potential  $B_\mu$  is complex,  $\alpha = \alpha(x^\mu) = \beta + i\delta$ ,  $\beta = \beta(x^\mu)$ ,  $\delta = \delta(x^\mu)$ , and  $\beta$ ,  $\delta$  are real. The imaginary part of the complex four-potential is a gradient of a certain function, so alternatively we can use this function instead of the imaginary components of the four-potential.

After the transform, the equations of spinor electrodynamics can be rewritten as follows:

$$(i\cancel{\partial} - \cancel{B})\varphi = \varphi, \quad (6)$$

$$\square B_\mu - B_{,\nu\mu}^\nu = \exp(-2\delta) e^2 \bar{\varphi} \gamma_\mu \varphi. \quad (7)$$

If  $\psi$  and  $\varphi$  have components

$$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{pmatrix}, \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}, \quad (8)$$

let us fix the "gauge transform" of Eqs.(4,5) somewhat arbitrarily by the following condition:

$$\varphi_1 = \exp(-i\alpha)\psi = 1. \quad (9)$$

The Dirac equation (6) can be written in components as follows:

$$(B^0 + B^3)\varphi_3 + (B^1 - iB^2)\varphi_4 + i(\varphi_{3,3} - i\varphi_{4,2} + \varphi_{4,1} - \varphi_{3,0}) = \varphi_1, \quad (10)$$

$$(B^1 + iB^2)\varphi_3 + (B^0 - B^3)\varphi_4 - i(\varphi_{4,3} - i\varphi_{3,2} - \varphi_{3,1} + \varphi_{4,0}) = \varphi_2, \quad (11)$$

$$(B^0 - B^3)\varphi_1 - (B^1 - iB^2)\varphi_2 - i(\varphi_{1,3} - i\varphi_{2,2} + \varphi_{2,1} + \varphi_{1,0}) = \varphi_3, \quad (12)$$

$$- (B^1 + iB^2)\varphi_1 + (B^0 + B^3)\varphi_2 + i\varphi_{2,3} + \varphi_{1,2} - i(\varphi_{1,1} + \varphi_{2,0}) = \varphi_4. \quad (13)$$

Equations (12,13) can be used to express components  $\varphi_3, \varphi_4$  via  $\varphi_1, \varphi_2$  and eliminate them from equations (10,11). The resulting equations for  $\varphi_1$  and  $\varphi_2$  are as follows:

$$\begin{aligned} & -\varphi_{1,\mu}^\mu + \varphi_2(-iB_{,3}^1 - B_{,3}^2 + B_{,2}^0 + B_{,2}^3 + i(B_{,1}^0 + B_{,1}^3 + B_{,0}^1) + B_{,0}^2) + \\ & + \varphi_1(-1 + B^\mu B_\mu - iB_{,\mu}^\mu + iB_{,3}^0 - B_{,2}^1 + B_{,1}^2 + iB_{,0}^3) - 2iB^\mu \varphi_{1,\mu} = 0, \end{aligned} \quad (14)$$

$$\begin{aligned} & -\varphi_{2,\mu}^\mu + i\varphi_1(B_{,3}^1 + iB_{,3}^2 + iB_{,2}^0 - iB_{,2}^3 + B_{,1}^0 - B_{,1}^3 + B_{,0}^1 + iB_{,0}^2) + \\ & + \varphi_2(-1 + B^\mu B_\mu - i(B_{,\mu}^\mu + B_{,3}^0 + iB_{,2}^1 - iB_{,1}^2 + B_{,0}^3)) - 2iB^\mu \varphi_{2,\mu} = 0. \end{aligned} \quad (15)$$

Equation (14) can be used to express  $\varphi_2$  via  $\varphi_1$ :

$$\varphi_2 = - (iF^1 + F^2)^{-1} (\square' + iF^3) \varphi_1, \quad (16)$$

where  $F^i = E^i + iH^i$ , real electric field  $E^i$  and magnetic field  $H^i$  are defined by the standard formulas

$$F^{\mu\nu} = B^{\nu,\mu} - B^{\mu,\nu} = \begin{pmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -H^3 & H^2 \\ E^2 & H^3 & 0 & -H^1 \\ E^3 & -H^2 & H^1 & 0 \end{pmatrix}, \quad (17)$$

and the modified d'Alembertian  $\square'$  is defined as follows:

$$\square' = \partial^\mu \partial_\mu + 2iB^\mu \partial_\mu + iB_{,\mu}^\mu - B^\mu B_\mu + 1. \quad (18)$$

Equation (15) can be rewritten as follows:

$$-(\square' - iF^3) \varphi_2 - (iF^1 - F^2) \varphi_1 = 0. \quad (19)$$

Substitution of  $\varphi_2$  from equation (16) into equation (15) yields an equation of the fourth order for  $\varphi_1$ :

$$\left( (\square' - iF^3) (iF^1 + F^2)^{-1} (\square' + iF^3) - iF^1 + F^2 \right) \varphi_1 = 0. \quad (20)$$

Application of the gauge condition of Eq.(9) to Eqs.(18,16,20, and 19) yields the following equations:

$$\square' \varphi_1 = iB_{,\mu}^\mu - B^\mu B_\mu + 1, \quad (21)$$

$$\varphi_2 = -(iF^1 + F^2)^{-1} (iB_{,\mu}^\mu - B^\mu B_\mu + 1 + iF^3), \quad (22)$$

$$(\square' - iF^3) (iF^1 + F^2)^{-1} (iB_{,\mu}^\mu - B^\mu B_\mu + 1 + iF^3) - iF^1 + F^2 = 0, \quad (23)$$

$$-(\square' - iF^3) \varphi_2 - (iF^1 - F^2) \varphi_1 = 0. \quad (24)$$

Obviously, Eqs.(9,22,12, and 13) can be used to eliminate spinor  $\varphi$  from the equations of spinor electrodynamics (6,7). It is then possible to eliminate  $\delta$  from the resulting equations. Furthermore, it turns out that the equations describe independent dynamics of the (complex four-potential of) electromagnetic field  $B^\mu$ . More precisely, if components  $B^\mu$  and their temporal derivatives (derivatives with respect to  $x^0$ ) up to the second order  $\dot{B}^\mu$  and  $\ddot{B}^\mu$  are known at some point in time in the entire 3D space  $x^0=\text{const}$ , the equations determine the temporal derivatives of the third order  $\ddot{\ddot{B}}^\mu$ , so the Cauchy problem can be posed, and the equations can be integrated (at least locally). Let us prove this statement.

### III. INDEPENDENT EVOLUTION OF THE ELECTROMAGNETIC FIELD

As  $\varphi_1=1$  (Eq.(9)), we obtain

$$\bar{\varphi}\gamma_\mu\varphi = \begin{pmatrix} \varphi_1^*\varphi_1 + \varphi_2^*\varphi_2 + \varphi_3^*\varphi_3 + \varphi_4^*\varphi_4 \\ \varphi_2^*\varphi_1 + \varphi_1^*\varphi_2 - \varphi_4^*\varphi_3 - \varphi_3^*\varphi_4 \\ i\varphi_2^*\varphi_1 - i\varphi_1^*\varphi_2 - i\varphi_4^*\varphi_3 + i\varphi_3^*\varphi_4 \\ \varphi_1^*\varphi_1 - \varphi_2^*\varphi_2 - \varphi_3^*\varphi_3 + \varphi_4^*\varphi_4 \end{pmatrix} = \begin{pmatrix} 1 + \varphi_2^*\varphi_2 + \varphi_3^*\varphi_3 + \varphi_4^*\varphi_4 \\ \varphi_2^* + \varphi_2 - \varphi_4^*\varphi_3 - \varphi_3^*\varphi_4 \\ i\varphi_2^* - i\varphi_2 - i\varphi_4^*\varphi_3 + i\varphi_3^*\varphi_4 \\ 1 - \varphi_2^*\varphi_2 - \varphi_3^*\varphi_3 + \varphi_4^*\varphi_4 \end{pmatrix}. \quad (25)$$

Using Eq.(7) with index  $\mu = 0$  and Eq.(25), we can express  $e^2 \exp(-2\delta)$  as follows:

$$e^2 \exp(-2\delta) = (B_{0,i}^i - B_{,i0}^i) (1 + \varphi_2^*\varphi_2 + \varphi_3^*\varphi_3 + \varphi_4^*\varphi_4)^{-1}, \quad (26)$$

as

$$\square B_0 - B_{,\nu 0}^\nu = B_{0,i}^i - B_{,i0}^i \quad (27)$$

(Latin indices run from 1 to 3, and Greek indices run from 0 to 3). Substitution of Eq.(26) in Eq.(7) yields

$$\square B_i - B_{,\nu i}^\nu = \ddot{B}_i + B_{i,j}^j - \dot{B}_{,i}^0 - B_{,ji}^j = (B_{0,j}^j - B_{,j0}^j) (1 + \varphi_2^*\varphi_2 + \varphi_3^*\varphi_3 + \varphi_4^*\varphi_4)^{-1} \begin{pmatrix} \varphi_2^* + \varphi_2 - \varphi_4^*\varphi_3 - \varphi_3^*\varphi_4 \\ i\varphi_2^* - i\varphi_2 - i\varphi_4^*\varphi_3 + i\varphi_3^*\varphi_4 \\ 1 - \varphi_2^*\varphi_2 - \varphi_3^*\varphi_3 + \varphi_4^*\varphi_4 \end{pmatrix}. \quad (28)$$

We note based on Eq.(22) that  $\varphi_2$  can be expressed via  $B^\mu$ ,  $\dot{B}^\mu$ , and their spatial derivatives (derivatives with respect to  $x^1$ ,  $x^2$ , and  $x^3$ ), as

$$F^1 = E^1 + iH^1 = F^{10} + iF^{32} = B^{0,1} - B^{1,0} + i(B^{2,3} - B^{3,2}), \quad (29)$$

$$F^2 = E^2 + iH^2 = F^{20} + iF^{13} = B^{0,2} - B^{2,0} + i(B^{3,1} - B^{1,3}), \quad (30)$$

$$F^3 = E^3 + iH^3 = F^{30} + iF^{21} = B^{0,3} - B^{3,0} + i(B^{1,2} - B^{2,1}). \quad (31)$$

Using Eqs.(29,30,31), the first temporal derivatives of  $F^i$  can be written as follows:

$$\dot{F}^1 = \dot{B}^{0,1} - \ddot{B}^1 + i(\dot{B}^{2,3} - \dot{B}^{3,2}), \quad (32)$$

$$\dot{F}^2 = \dot{B}^{0,2} - \ddot{B}^2 + i(\dot{B}^{3,1} - \dot{B}^{1,3}), \quad (33)$$

$$\dot{F}^3 = \dot{B}^{0,3} - \ddot{B}^3 + i(\dot{B}^{1,2} - \dot{B}^{2,1}). \quad (34)$$

We note based on Eq.(22,29,30,31,32,33,34) that  $\dot{\varphi}_2$  can be expressed via  $B^\mu$ ,  $\dot{B}^\mu$ ,  $\ddot{B}^\mu$ , and their spatial derivatives.

From Eqs.(12,9) we obtain:

$$\varphi_3 = B^0 - B^3 - (B^1 - iB^2)\varphi_2 - i(-i\varphi_{2,2} + \varphi_{2,1}). \quad (35)$$

We note that that  $\varphi_3$  can be expressed via  $B^\mu$ ,  $\dot{B}^\mu$ , and their spatial derivatives. The first temporal derivative of  $\varphi_3$  can be written as follows:

$$\dot{\varphi}_3 = \dot{B}^0 - \dot{B}^3 - (\dot{B}^1 - i\dot{B}^2)\varphi_2 - (B^1 - iB^2)\dot{\varphi}_2 - i(-i\dot{\varphi}_{2,2} + \dot{\varphi}_{2,1}). \quad (36)$$

We note based on Eq.(36) that  $\dot{\varphi}_3$  can be expressed via  $B^\mu$ ,  $\dot{B}^\mu$ ,  $\ddot{B}^\mu$ , and their spatial derivatives.

From Eqs.(13,9) we obtain:

$$\varphi_4 = -(B^1 + iB^2) + (B^0 + B^3)\varphi_2 + i\varphi_{2,3} - i\varphi_{2,0}. \quad (37)$$

We note that  $\varphi_4$  can be expressed via  $B^\mu$ ,  $\dot{B}^\mu$ ,  $\ddot{B}^\mu$ , and their spatial derivatives. The first temporal derivative of  $\varphi_4$  can be written as follows:

$$\dot{\varphi}_4 = -(\dot{B}^1 + i\dot{B}^2) + (\dot{B}^0 + \dot{B}^3)\varphi_2 + (B^0 + B^3)\dot{\varphi}_2 + i\dot{\varphi}_{2,3} - i\ddot{\varphi}_2. \quad (38)$$

All terms in the expression for  $\dot{\varphi}_4$  with a possible exception of  $-i\ddot{\varphi}_2$  can be expressed via  $B^\mu$ ,  $\dot{B}^\mu$ ,  $\ddot{B}^\mu$ , and their spatial derivatives. Let us consider the expression  $\ddot{\varphi}_2$ .

Eqs.(24,18) yield:

$$\begin{aligned} 0 &= -(\square' - iF^3)\varphi_2 - (iF^1 - F^2) = \\ &- (\partial^\mu \partial_\mu + 2iB^\mu \partial_\mu + iB_{,\mu}^\mu - B^\mu B_\mu + 1 - iF^3)\varphi_2 - (iF^1 - F^2) = \\ &- (\partial^0 \partial_0 + \partial^i \partial_i + 2iB^0 \partial_0 + 2iB^i \partial_i + iB_{,\mu}^\mu - B^\mu B_\mu + 1 - iF^3)\varphi_2 - (iF^1 - F^2) = \\ &- \ddot{\varphi}_2 - 2iB^0 \dot{\varphi}_2 - (\partial^i \partial_i + 2iB^i \partial_i + iB_{,\mu}^\mu - B^\mu B_\mu + 1 - iF^3)\varphi_2 - (iF^1 - F^2). \end{aligned} \quad (39)$$

We note that  $\ddot{\varphi}_2$  can be expressed via  $B^\mu$ ,  $\dot{B}^\mu$ ,  $\ddot{B}^\mu$ , and their spatial derivatives. Therefore, based on Eqs.(38), the same is true for  $\dot{\varphi}_4$ . Furthermore, we can summarize that all functions  $\varphi_\mu$  and  $\dot{\varphi}_\mu$  can be expressed via  $B^\mu$ ,  $\dot{B}^\mu$ ,  $\ddot{B}^\mu$ , and their spatial derivatives. Obviously, the same is true for  $\varphi_\mu^*$  and  $\dot{\varphi}_\mu^*$ .

Differentiating Eqs.(28) with respect to time ( $x^0$ ), we conclude that functions  $\ddot{B}^i$  can be expressed via  $B^\mu$ ,  $\dot{B}^\mu$ ,  $\ddot{B}^\mu$ , and their spatial derivatives, as the left-hand side of Eqs.(28) after the differentiation equals

$$\ddot{B}_i + \dot{B}_{i,j}^j - \ddot{B}_{,i}^0 - \dot{B}_{,ji}^j, \quad (40)$$

and the right-hand side of Eq.(28) after the differentiation will be expressed via  $B^\mu$ ,  $\dot{B}^\mu$ ,  $\ddot{B}^\mu$ ,  $\varphi_\mu$ ,  $\dot{\varphi}_\mu$ ,  $\varphi_\mu^*$ ,  $\dot{\varphi}_\mu^*$ , and their spatial derivatives. Therefore, functions  $\ddot{B}_i$  can be expressed via  $B^\mu$ ,  $\dot{B}^\mu$ ,  $\ddot{B}^\mu$ , and their spatial derivatives, so to prove the initial statement we just need to prove the same for  $\ddot{B}_0$ . To this end, let us consider the following equation derived from Eqs.(23,18):

$$\begin{aligned} & (\partial^\mu \partial_\mu + 2iB^\mu \partial_\mu + iB_{,\mu}^\mu - B^\mu B_\mu + 1 - iF^3) (iF^1 + F^2)^{-1} (iB_{,\mu}^\mu - B^\mu B_\mu + 1 + iF^3) - \\ & iF^1 + F^2 = 0. \end{aligned} \quad (41)$$

It is obvious that the following part of the left-hand side of Eq.(41) can be expressed via  $B^\mu$ ,  $\dot{B}^\mu$ , and their spatial derivatives:

$$(iB_{,\mu}^\mu - B^\mu B_\mu + 1 - iF^3) (iF^1 + F^2)^{-1} (iB_{,\mu}^\mu - B^\mu B_\mu + 1 + iF^3) - iF^1 + F^2. \quad (42)$$

The rest of the left-hand side of Eq.(41) can be rewritten as follows:

$$(\partial^0 \partial_0 + \partial^i \partial_i + 2iB^0 \partial_0 + 2iB^i \partial_i) (iF^1 + F^2)^{-1} (iB_{,\mu}^\mu - B^\mu B_\mu + 1 + iF^3). \quad (43)$$

The following part of the expression in Eq.(43) can be expressed via  $B^\mu$ ,  $\dot{B}^\mu$ , and their spatial derivatives:

$$(\partial^i \partial_i + 2iB^i \partial_i) (iF^1 + F^2)^{-1} (iB_{,\mu}^\mu - B^\mu B_\mu + 1 + iF^3). \quad (44)$$

Let us evaluate the following expression:

$$\begin{aligned} & \partial_0 (iF^1 + F^2)^{-1} (iB_{,\mu}^\mu - B^\mu B_\mu + 1 + iF^3) = \\ & - \left( i\dot{F}^1 + \dot{F}^2 \right) (iF^1 + F^2)^{-2} (iB_{,\mu}^\mu - B^\mu B_\mu + 1 + iF^3) + \\ & (iF^1 + F^2)^{-1} \left( i\dot{B}_{,\mu}^\mu - 2\dot{B}^\mu B_\mu + i\dot{F}^3 \right). \end{aligned} \quad (45)$$

Thus, the term  $2iB^0 \partial_0$  in the first pair of parentheses of Eq.(43) produces terms that can be expressed via  $B^\mu$ ,  $\dot{B}^\mu$ ,  $\ddot{B}^\mu$ , and their spatial derivatives. Therefore, we only need to evaluate



(using Eq.(45)) the following expression:

$$\begin{aligned}
& \partial^0 \partial_0 (iF^1 + F^2)^{-1} (iB_{,\mu}^\mu - B^\mu B_\mu + 1 + iF^3) = \\
& \partial^0 \left( - (i\dot{F}^1 + \dot{F}^2) (iF^1 + F^2)^{-2} (iB_{,\mu}^\mu - B^\mu B_\mu + 1 + iF^3) \right) + \\
& \partial^0 (iF^1 + F^2)^{-1} (i\dot{B}_{,\mu}^\mu - 2\dot{B}^\mu B_\mu + i\dot{F}^3) = \\
& \partial^0 \left( - (i\dot{F}^1 + \dot{F}^2) (iF^1 + F^2)^{-2} (iB_{,\mu}^\mu - B^\mu B_\mu + 1 + iF^3) \right) + \\
& \left( \partial^0 (iF^1 + F^2)^{-1} \right) (i\dot{B}_{,\mu}^\mu - 2\dot{B}^\mu B_\mu + i\dot{F}^3) + \\
& (iF^1 + F^2)^{-1} \left( i\ddot{B}^0 + i\ddot{B}_{,i}^i + \left( \partial^0 (-2\dot{B}^\mu B_\mu + i\dot{F}^3) \right) \right).
\end{aligned} \tag{46}$$

It follows from Eqs.(32,33,34) that  $\ddot{F}^i$  can be expressed via  $B^\mu$ ,  $\dot{B}^\mu$ ,  $\ddot{B}^\mu$ ,  $\ddot{B}^i$  (but not  $\ddot{B}^0$ ), and their spatial derivatives, but, as explained above,  $\ddot{B}^i$  can be expressed via  $B^\mu$ ,  $\dot{B}^\mu$ ,  $\ddot{B}^\mu$ , and their spatial derivatives. Thus, this is also true for all terms of Eq.(46) (and, consequently, Eq.(41)), with a possible exception of the term

$$(iF^1 + F^2)^{-1} \ddot{B}^0, \tag{47}$$

but that means that Eq.(41) can be used to express  $\ddot{B}^0$  via  $B^\mu$ ,  $\dot{B}^\mu$ ,  $\ddot{B}^\mu$ , and their spatial derivatives, which completes the proof.

#### IV. LINEARIZATION

Based on the equations of independent evolution of electromagnetic field, a system of linear equations in the Fock space can be built, which looks like a second-quantized theory and is equivalent to the original nonlinear system on the set of solutions of the latter. The approach of Ref. [1] is adopted virtually without any modifications. This approach is based on nightlight's observation (Ref. [7]) that a generalization of the Carleman linearization procedure (Ref. [8]) can be used for this purpose.

Let us consider a nonlinear differential equation in an  $(s+1)$ -dimensional space-time (the equations describing independent dynamics of electromagnetic field for spinor electrodynamics are a special case of this equation)  $\partial_t \boldsymbol{\xi}(x, t) = \mathbf{F}(\boldsymbol{\xi}, D^\alpha \boldsymbol{\xi}; x, t)$ ,  $\boldsymbol{\xi}(x, 0) = \boldsymbol{\xi}_0(x)$ , where  $\boldsymbol{\xi} : \mathbf{R}^s \times \mathbf{R} \rightarrow \mathbf{C}^k$  (in our case,  $\boldsymbol{\xi}$  includes real and imaginary parts of  $B^\mu$ ,  $\dot{B}^\mu$ , and  $\ddot{B}^\mu$ ),  $D^\alpha \boldsymbol{\xi} = (D^{\alpha_1} \xi_1, \dots, D^{\alpha_k} \xi_k)$ ,  $\alpha_i$  are multiindices,  $D^\beta = \partial^{|\beta|} / \partial x_1^{\beta_1} \dots \partial x_s^{\beta_s}$ , with  $|\beta| = \sum_{i=1}^s \beta_i$ , is a generalized derivative,  $\mathbf{F}$  is analytic in  $\boldsymbol{\xi}$ ,  $D^\alpha \boldsymbol{\xi}$ . It is also assumed that  $\boldsymbol{\xi}_0$  and  $\boldsymbol{\xi}$  are square

integrable. Then Bose operators  $\mathbf{a}^\dagger(\mathbf{x}) = (a_1^\dagger(x), \dots, a_k^\dagger(x))$  and  $\mathbf{a}(\mathbf{x}) = (a_1(x), \dots, a_k(x))$  are introduced with the canonical commutation relations:

$$\begin{aligned} [a_i(x), a_j^\dagger(x')] &= \delta_{ij} \delta(x - x') I, \\ [a_i(x), a_j(x')] &= [a_i^\dagger(x), a_j^\dagger(x')] = 0, \end{aligned} \quad (48)$$

where  $x, x' \in \mathbf{R}^s$ ,  $i, j = 1, \dots, k$ . Normalized functional coherent states in the Fock space are defined as  $|\boldsymbol{\xi}\rangle = \exp\left(-\frac{1}{2} \int d^s x |\boldsymbol{\xi}|^2\right) \exp\left(\int d^s x \boldsymbol{\xi}(x) \cdot \mathbf{a}^\dagger(x)\right) |\mathbf{0}\rangle$ . They have the following property:

$$\mathbf{a}(x)|\boldsymbol{\xi}\rangle = \boldsymbol{\xi}(x)|\boldsymbol{\xi}\rangle, \quad (49)$$

. Then the following vectors in the Fock space can be introduced:

$$\begin{aligned} |\boldsymbol{\xi}, t\rangle &= \exp\left[\frac{1}{2} \left(\int d^s x |\boldsymbol{\xi}|^2 - \int d^s x |\boldsymbol{\xi}_0|^2\right)\right] |\boldsymbol{\xi}\rangle \\ &= \exp\left(-\frac{1}{2} \int d^s x |\boldsymbol{\xi}_0|^2\right) \\ &\quad \times \exp\left(\int d^s x \boldsymbol{\xi}(x) \cdot \mathbf{a}^\dagger(x)\right) |\mathbf{0}\rangle. \end{aligned} \quad (50)$$

Differentiation of Eq. (50) with respect to time  $t$  yields, together with Eq. (49), a linear Schrödinger-like evolution equation in the Fock space:

$$\begin{aligned} \frac{d}{dt} |\boldsymbol{\xi}, t\rangle &= M(t) |\boldsymbol{\xi}, t\rangle, \\ |\boldsymbol{\xi}, 0\rangle &= |\boldsymbol{\xi}_0\rangle, \end{aligned} \quad (51)$$

where the boson "Hamiltonian"  $M(t) = \int d^s x \mathbf{a}^\dagger(x) \cdot F(\mathbf{a}(x), D^\alpha \mathbf{a}(x))$ .

A question can arise: can this procedure be applied to spinor electrodynamics directly, without elimination of the spinor field? It seems however that the procedure is better suited to bosonic fields.

## V. CONCLUSION

The results of Ref. [1] are generalized to spinor electrodynamics: after a generalized gauge transform yielding a complex electromagnetic four-potential, but preserving the electromagnetic fields, the spinor field is eliminated from the equations of spinor electrodynamics, and the resulting system of equations for the complex four-potential is proven to describe independent evolution of the electromagnetic field. Application of a generalized Carleman

linearization procedure to the system yields a system of linear equations in the Fock space that looks like a second-quantized theory, but is equivalent to the initial system on the set of solutions of the latter. While comparison with standard quantum electrodynamics and experimental data has not been performed, this work presents new constructive results for a realistic theory (spinor electrodynamics) and may be important for interpretation of quantum theory.

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